Ising and Heisenberg models

A simplified model of magnetic materials

A playground for studying phase transitions

$$H = -\sum_{(ij)} J_{ij}\, s_i s_j - h \sum_i s_i$$

(Classical) spins $s_i = \pm 1$

magnetic field

$J_{ij} > 0$ – <u>ferromagnetic</u> interactions

$J_{ij} < 0$ – <u>antiferromagnetic</u> interaction

(Invented by Wilhelm Lenz; Ising studied it in his PhD thesis and solved the 1D case)

- 1D and 2D cases are doable exactly
- one may study phase transitions and the structure of (the ground) state (especially in the case of AFM interactions)

Heisenberg model

$$\hat{H} = -\sum_{(ij)} J_{ij}\, \hat{S}_i \hat{S}_j - h \sum_i \hat{S}_i$$

$\hat{S}_i$ – quantum (3D) spin

(There may be other terms or anisotropy, it is still called Heisenberg model)

Quantum Ising model

$$\hat{H} = -\sum_{(ij)} J_{ij}\, \hat{S}_i^z \hat{S}_j^z - h \sum_i \hat{S}_i$$

be mapped

$$\hat{H} = -\sum_{(ij)} J_{ij} S_i^z S_j^z - h \sum_i S_i$$

For $\hbar = 0$ that quantum model may be mapped onto the classical Ising model because $S_i = \pm \frac{1}{2}$ are the eigenstates

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In a finite volume, the Ising model does not have a phase transition because $e^{-\beta E}$ is an analytic function and so is $\sum_i e^{-\beta E_i}$.